Linear-programming Decoding of Nonbinary Linear Codes

Mark F. Flanagan
Institute for Digital Communications, University of Edinburgh

Vitaly Skachek, Eimear Byrne, Marcus Greferath
Claude Shannon Institute, Dublin, Ireland

IDCOM Signal & Image Processing Joint Seminar Programme
January 8, 2008
Motivation

Binary LDPC codes:

- Very good error-correcting performance.
Motivation

Binary LDPC codes:

- Very good error-correcting performance.
- Difficult to analyze/optimize.
Motivation

Binary LDPC codes:

- Very good error-correcting performance.
- Difficult to analyze/optimize.
- [Feldman Wainwright Karger ’03–’05] - linear-programming decoding has good performance with analytical results
Motivation

Binary LDPC codes:

- Very good error-correcting performance.
- Difficult to analyze/optimize.
- [Feldman Wainwright Karger ’03–’05] - linear-programming decoding has good performance with analytical results

LDPC Coded Modulation:

- Bit-interleaved coded modulation (BICM) popular with LDPC codes
Motivation

Binary LDPC codes:

- Very good error-correcting performance.
- Difficult to analyze/optimize.
- [Feldman Wainwright Karger ’03–’05] - linear-programming decoding has *good performance* with *analytical results*

LDPC Coded Modulation:

- Bit-interleaved coded modulation (BICM) popular with LDPC codes
- Presents analytical difficulties
Motivation

Binary LDPC codes:
- Very good error-correcting performance.
- Difficult to analyze/optimize.
- [Feldman Wainwright Karger '03–'05] - linear-programming decoding has *good performance with analytical results*

LDPC Coded Modulation:
- Bit-interleaved coded modulation (BICM) popular with LDPC codes
- Presents analytical difficulties
- Alternative - *nonbinary* codes whose symbols map directly to modulation signals
In this work we generalize the framework and theorems in
[Feldman Wainwright Karger ’05] to nonbinary codes.
In this work we generalize the framework and theorems in [Feldman Wainwright Karger ’05] to nonbinary codes.

- Nonbinary LP decoding problem formulated.
In this work we generalize the framework and theorems in [Feldman Wainwright Karger ’05] to nonbinary codes.

- Nonbinary LP decoding problem formulated.
- Study of properties of LP decoding in nonbinary case.
In this work we generalize the framework and theorems in [Feldman Wainwright Karger ’05] to nonbinary codes.

- Nonbinary LP decoding problem formulated.
- Study of properties of LP decoding in nonbinary case.
- Link between pseudocodeword concepts.
In this work we generalize the framework and theorems in [Feldman Wainwright Karger ’05] to nonbinary codes.

- Nonbinary LP decoding problem formulated.
- Study of properties of LP decoding in nonbinary case.
- Link between pseudocodeword concepts.
- Channel symmetry condition for codeword-independent performance.
In this work we generalize the framework and theorems in [Feldman Wainwright Karger ’05] to nonbinary codes.

- Nonbinary LP decoding problem formulated.
- Study of properties of LP decoding in nonbinary case.
- Link between pseudocodeword concepts.
- Channel symmetry condition for codeword-independent performance.
- Simulation-based comparison with ML Decoding.
Denote: \( \mathcal{R} \) a ring with \( q \) elements, 0 its additive identity, \( \mathcal{R}^- = \mathcal{R}\{0\} \).

Let \( \mathcal{C} \) be a linear code with \( m \times n \) parity-check matrix \( \mathbf{H} \) over \( \mathcal{R} \).

If \( I = \{1, 2, \ldots, n\} \) and \( J = \{1, 2, \ldots, m\} \) are the sets of column and row indices, respectively. For each \( j \in J \), we denote by \( I_j \) the set of non-zero positions in the \( j \)-th row of \( \mathbf{H} \).

\( d \) denotes the length of the 'longest' parity check.

Example:
\[ \mathcal{R} = \mathbb{Z}_3, \quad \mathbf{H} = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix}, \quad I_2 = \{1, 3, 4\}, \quad d = 4 \]
Denote: $\mathcal{R}$ a ring with $q$ elements, 0 its additive identity, $\mathcal{R}^- = \mathcal{R} \setminus \{0\}$.

Let $\mathcal{C}$ be a linear code with $m \times n$ parity-check matrix $\mathcal{H}$ over $\mathcal{R}$.

$d$ denotes the length of the 'longest' parity check.

Example $\mathcal{R} = \mathbb{Z}_3$, $\mathcal{H} = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix}$, $I_2 = \{1, 3, 4\}$, $d = 4$.
Denote: $\mathbb{K}$ a ring with $q$ elements, 0 its additive identity, $\mathbb{K}^\cdot = \mathbb{K}\setminus\{0\}$.

Let $C$ be a linear code with $m \times n$ parity-check matrix $H$ over $\mathbb{K}$.

$I = \{1, 2, \cdots, n\}$ and $J = \{1, 2, \cdots, m\}$ are the sets of column and row indices, respectively.

d denotes the length of the 'longest' parity check.

Example $\mathbb{R} = \mathbb{Z}_3$, $H = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix}$, $I_2 = \{1, 3, 4\}$, $d = 4$. 
Denote: $\mathcal{R}$ a ring with $q$ elements, 0 its additive identity, $\mathcal{R}^- = \mathcal{R}\setminus\{0\}$.

Let $\mathcal{C}$ be a linear code with $m \times n$ parity-check matrix $H$ over $\mathcal{R}$.

$I = \{1, 2, \cdots, n\}$ and $J = \{1, 2, \cdots, m\}$ are the sets of column and row indices, respectively.

For each $j \in J$, we denote by $I_j$ the set of non-zero positions in the $j$-th row of $H$.
Denote: \( \mathbb{R} \) a ring with \( q \) elements, 0 its additive identity, \( \mathbb{R}^- = \mathbb{R}\{0\} \).

Let \( \mathcal{C} \) be a linear code with \( m \times n \) parity-check matrix \( \mathcal{H} \) over \( \mathbb{R} \).

\( I = \{1, 2, \cdots, n\} \) and \( J = \{1, 2, \cdots, m\} \) are the sets of column and row indices, respectively.

For each \( j \in J \), we denote by \( I_j \) the set of non-zero positions in the \( j \)-th row of \( \mathcal{H} \).

\( d \) denotes the length of the ‘longest’ parity check.
Denote: \( \mathcal{R} \) a ring with \( q \) elements, 0 its additive identity, \( \mathcal{R}^{-} = \mathcal{R}\setminus\{0\} \).

Let \( \mathcal{C} \) be a linear code with \( m \times n \) parity-check matrix \( \mathcal{H} \) over \( \mathcal{R} \).

\( \mathcal{I} = \{1, 2, \cdots, n\} \) and \( \mathcal{J} = \{1, 2, \cdots, m\} \) are the sets of column and row indices, respectively.

For each \( j \in \mathcal{J} \), we denote by \( \mathcal{I}_j \) the set of non-zero positions in the \( j \)-th row of \( \mathcal{H} \).

\( d \) denotes the length of the ‘longest’ parity check.

Example

\[ \mathcal{R} = \mathbb{Z}_3, \quad \mathcal{H} = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix}, \quad \mathcal{I}_2 = \{1, 3, 4\}, \quad d = 4 \]
For $c \in \mathcal{R}^n$, parity check $j \in J$ is satisfied by $c$ iff

$$\sum_{i \in I_j} H_{j,i} \cdot c_i = 0.$$
For $c \in \mathbb{R}^n$, parity check $j \in \mathcal{J}$ is *satisfied* by $c$ iff

$$\sum_{i \in I_j} H_{j,i} \cdot c_i = 0.$$ 

We define the projection mapping for $j \in \mathcal{J}$ by

$$x_j(c) = (c_i)_{i \in I_j}.$$
For $c \in \mathbb{R}^n$, parity check $j \in \mathcal{J}$ is satisfied by $c$ iff
\[ \sum_{i \in \mathcal{I}_j} H_{j,i} \cdot c_i = 0. \]

We define the projection mapping for $j \in \mathcal{J}$ by
\[ x_j(c) = (c_i)_{i \in \mathcal{I}_j} \]

Thus, parity check $j \in \mathcal{J}$ is satisfied by $c$ iff
\[ x_j(c) \in C_j \]
Communication Model

The codeword \( \tilde{c} = (\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n) \in \mathcal{C} \) has been transmitted over a \( q \)-ary input memoryless channel.

A corrupted word \( y = (y_1, y_2, \cdots, y_n) \in \Sigma \) has been received. \( \Sigma \) denotes the set of channel output symbols. This \( \Sigma \) either has finite cardinality, or is equal to \( \mathbb{R}^l \) or \( \mathbb{C}^l \) for some integer \( l \geq 1 \).

Assume that all information words are equally probable.
The codeword $\bar{c} = (\bar{c}_1, \bar{c}_2, \cdots, \bar{c}_n) \in \mathcal{C}$ has been transmitted over a $q$-ary input memoryless channel.

A corrupted word $y = (y_1, y_2, \cdots, y_n) \in \Sigma^n$ has been received. $\Sigma$ denotes the set of channel output symbols.
The codeword \( \bar{c} = (\bar{c}_1, \bar{c}_2, \cdots, \bar{c}_n) \in \mathcal{C} \) has been transmitted over a \( q \)-ary input memoryless channel.

A corrupted word \( \bar{y} = (y_1, y_2, \cdots, y_n) \in \Sigma^n \) has been received. \( \Sigma \) denotes the set of channel output symbols.

This \( \Sigma \) either has finite cardinality, or is equal to \( \mathbb{R}^l \) or \( \mathbb{C}^l \) for some integer \( l \geq 1 \).
Communication Model

- The codeword $\bar{c} = (\bar{c}_1, \bar{c}_2, \cdots, \bar{c}_n) \in \mathcal{C}$ has been transmitted over a $q$-ary input memoryless channel.
- A corrupted word $y = (y_1, y_2, \cdots, y_n) \in \Sigma^n$ has been received. $\Sigma$ denotes the set of channel output symbols.
- This $\Sigma$ either has finite cardinality, or is equal to $\mathbb{R}^l$ or $\mathbb{C}^l$ for some integer $l \geq 1$.
- Assume that all information words are equally probable.
Alphabet Mapping

The mapping

\[ \xi : \mathbb{R} \rightarrow \{0, 1\}^{q-1} \subset \mathbb{R}^{q-1}, \]

is defined by

\[ \xi(\alpha) = x = (x(\gamma))_{\gamma \in \mathbb{R}^-}, \]

such that, for each \( \gamma \in \mathbb{R}^- \),

\[ x(\gamma) = \begin{cases} 1 & \text{if } \gamma = \alpha \\ 0 & \text{otherwise}. \end{cases} \]
The mapping

\[ \xi : \mathbb{R} \rightarrow \{0, 1\}^{q-1} \subset \mathbb{R}^{q-1} , \]

is defined by

\[ \xi(\alpha) = x = (x(\gamma))_{\gamma \in \mathbb{R}^-} , \]

such that, for each \( \gamma \in \mathbb{R}^- \),

\[ x(\gamma) = \begin{cases} 
1 & \text{if } \gamma = \alpha \\
0 & \text{otherwise.} 
\end{cases} \]

The mapping \( \xi(\cdot) \) is one-to-one. Its image is the set of binary vectors of length \( q - 1 \) with Hamming weight 0 or 1.
The mapping

\[ \xi : \mathbb{R} \rightarrow \{0, 1\}^{q-1} \subset \mathbb{R}^{q-1}, \]

is defined by

\[ \xi(\alpha) = x = (x^{(\gamma)})_{\gamma \in \mathbb{R}^{-}}, \]

such that, for each \( \gamma \in \mathbb{R}^{-} \),

\[ x^{(\gamma)} = \begin{cases} 
1 & \text{if } \gamma = \alpha \\
0 & \text{otherwise.} 
\end{cases} \]

The mapping \( \xi(\cdot) \) is one-to-one. Its image is the set of binary vectors of length \( q - 1 \) with Hamming weight 0 or 1.
Definition

\[ \Xi : \mathbb{R}^n \rightarrow \{0, 1\}^{(q-1)n} \subset \mathbb{R}^{(q-1)n}, \]

where

\[ \Xi(c) = (\xi(c_1) \mid \xi(c_2) \mid \cdots \mid \xi(c_n)) . \]
Definition

\[ \Xi : \mathbb{R}^n \rightarrow \{0, 1\}^{(q-1)n} \subset \mathbb{R}^{(q-1)n}, \]
where

\[ \Xi(c) = (\xi(c_1) \mid \xi(c_2) \mid \cdots \mid \xi(c_n)) . \]

This \( \Xi \) is also one-to-one.
Vector $f \in \mathbb{R}^{(q-1)n}$,

$$f = (f_1 \mid f_2 \mid \cdots \mid f_n).$$
Vector \( f \in \mathbb{R}^{(q-1)n} \),

\[ f = (f_1 \mid f_2 \mid \cdots \mid f_n). \]

Here

\[ \forall i \in I, \ f_i = (f_i^{(\alpha)})_{\alpha \in \mathbb{R}}. \]
For $y \in \Sigma$, define $\lambda(y)$ by

$$
\lambda(y) = (\lambda^{(\alpha)}(y))_{\alpha \in \mathbb{R}^-},
$$

where, for each $y \in \Sigma$, $\alpha \in \mathbb{R}^-$,

$$
\lambda^{(\alpha)}(y) = \log \left( \frac{p(y|0)}{p(y|\alpha)} \right),
$$

and $p(y|c)$ denotes the channel output probability (density) conditioned on the channel input.
For $y \in \Sigma$, define $\lambda(y)$ by

$$\lambda(y) = (\lambda^{(\alpha)}(y))_{\alpha \in \mathbb{K}^{-}},$$

where, for each $y \in \Sigma$, $\alpha \in \mathbb{K}^{-}$,

$$\lambda^{(\alpha)}(y) = \log \left( \frac{p(y|0)}{p(y|\alpha)} \right),$$

and $p(y|c)$ denotes the channel output probability (density) conditioned on the channel input.

Extend $\lambda$ to a map on $\Sigma^n$ by

$$\lambda(y) = (\lambda(y_1) \mid \lambda(y_2) \mid \ldots \mid \lambda(y_n)).$$
MAXIMUM A POSTERIORI Decision Rule

\[
\hat{c} = \arg \max_{c \in C} p(\, c \mid y \,) \\
= \arg \max_{c \in C} \frac{p(\, y \mid c \,)p(\, c \,)}{p(\, y \,)}.
\]
Decision Rule

MAXIMUM A POSTERIORI Decision Rule

\[ \hat{c} = \arg \max_{c \in C} p(c | y) \]
\[ = \arg \max_{c \in C} \frac{p(y | c)p(c)}{p(y)}. \]

We may show

\[ \hat{c} = \arg \min_{c \in C} \lambda(y) \Xi(c)^T \]
Decision Rule over Convex Hull

\[ \hat{c} = \Xi^{-1}(\hat{f}) \]

where

\[ \hat{f} = \arg \min_{f \in K(\mathcal{C})} \lambda(y) f^T \]

\( \hat{c} \) represents the convex hull of all points \( f \in \mathbb{R}^{(q-1)n} \) which correspond to codewords.

The number of variables and constraints for this linear program is exponential in \( n \). To circumvent this problem, we formulate a relaxed LP problem.
Decision Rule over Convex Hull

\[ \hat{c} = \Xi^{-1}(\hat{f}) \]

where

\[ \hat{f} = \arg \min_{f \in K(C)} \lambda(y) f^T \]

\( K(C) \) represents the convex hull of all points \( f \in \mathbb{R}^{(q-1)n} \) which correspond to codewords.
\[ \hat{c} = \Xi^{-1}(\hat{f}) \]

where

\[ \hat{f} = \arg \min_{f \in \mathcal{K}(\mathcal{C})} \lambda(y)f^T \]

\( \mathcal{K}(\mathcal{C}) \) represents the convex hull of all points \( f \in \mathbb{R}^{(q-1)n} \) which correspond to codewords.

The number of variables and constraints for this linear program is exponential in \( n \). To circumvent this problem, we formulate a relaxed LP problem.
Auxiliary Variables

\[ w_{j,b} \quad \text{for} \quad j \in \mathcal{J}, \quad b \in C_j , \]
Auxiliary Variables

\[ w_{j,b} \text{ for } j \in \mathcal{J}, b \in \mathcal{C}_j, \]

The vector containing these variables:

\[ \mathbf{w} = \left( w_{j,b} \right)_{j \in \mathcal{J}, b \in \mathcal{C}_j}, \]

with respect to some ordering on the elements of \( \mathcal{C}_j \).
Relaxed LP Problem

Auxiliary Variables

\[ w_{j,b} \text{ for } j \in \mathcal{J}, b \in \mathcal{C}_j, \]

The vector containing these variables:

\[ \mathbf{w} = \left( w_{j,b} \right)_{j \in \mathcal{J}, b \in \mathcal{C}_j}, \]

with respect to some ordering on the elements of \( \mathcal{C}_j \).

The solution we seek for these variables is

\[ \forall j \in \mathcal{J} : w_{j,b} = \begin{cases} 1 & \text{if } b = \mathbf{x}_j(\bar{c}) \\ 0 & \text{otherwise} \end{cases}. \]
\[ \forall j \in J, \forall b \in C_j, \quad w_{j,b} \geq 0. \]
Linear Constraints

1. \( \forall j \in J, \forall b \in C_j, \quad w_{j,b} \geq 0 \).

2. \( \forall j \in J, \quad \sum_{b \in C_j} w_{j,b} = 1 \).
Linear Constraints

1. \( \forall j \in J, \forall b \in C_j, \quad w_{j,b} \geq 0 \). 

2. \( \forall j \in J, \sum_{b \in C_j} w_{j,b} = 1 \). 

3. \( \forall j \in J, \forall i \in I_j, \forall \alpha \in \mathbb{R}^- \), 
   \[ f_{i}^{(\alpha)} = \sum_{b \in C_j, b_i = \alpha} w_{j,b} \].
These constraints form a polytope which we denote by $Q$. 

If $\hat{f} \in \{0, 1\}^{(q-1)n}$, the output is the codeword $\Xi^{-1}(\hat{f})$.

Otherwise, the decoder outputs an 'error'.
These constraints form a polytope which we denote by $Q$.
The LP is defined by $O(qn + q^{d-1}m)$ variables and $O(qn + q^{d-1}m)$ constraints.
These constraints form a polytope which we denote by $Q$.
The LP is defined by $O(qn + q^{d-1}m)$ variables and $O(qn + q^{d-1}m)$ constraints.
If $\hat{f} \in \{0, 1\}^{(q-1)n}$, the output is the codeword $\Xi^{-1}(\hat{f})$. 
These constraints form a polytope which we denote by $Q$.
The LP is defined by $O(qn + q^{d-1}m)$ variables and $O(qn + q^{d-1}m)$ constraints.
If $\hat{f} \in \{0, 1\}^{(q-1)n}$, the output is the codeword $\Xi^{-1}(\hat{f})$.
Otherwise, the decoder outputs an ‘error’.
ML Certificate Property

Suppose that the decoder outputs a codeword $c \in \mathcal{C}$. Then, $c$ is the maximum-likelihood codeword.
Symmetry Condition

For each $\beta \in \mathbb{R}$, there exists a bijection

$$\tau_\beta : \Sigma \longrightarrow \Sigma,$$

such that the channel output probability (density) conditioned on the channel input satisfies

$$p(y|\alpha) = p(\tau_\beta(y)|\alpha - \beta),$$

for all $y \in \Sigma$, $\alpha \in \mathbb{R}$. When $\Sigma$ is equal to $\mathbb{R}^l$ or $\mathbb{C}^l$ for $l \geq 1$, the mapping $\tau_\beta$ is assumed to be isometric with respect to Euclidean distance in $\Sigma$, for every $\beta \in \mathbb{R}$. 

Theorem

Under the stated symmetry condition, the probability of decoder failure is independent of the transmitted codeword.

Examples:

- $q$-ary PSK over AWGN (with $R$ cyclic);
- orthogonal modulation over AWGN;
- discrete memoryless $q$-ary symmetric channel.

Mark F. Flanagan  
LP Decoding of Nonbinary Linear Codes
Symmetry Condition

For each $\beta \in \mathbb{R}$, there exists a bijection

$$\tau_\beta : \Sigma \longrightarrow \Sigma,$$

such that the channel output probability (density) conditioned on the channel input satisfies

$$p(y|\alpha) = p(\tau_\beta(y)|\alpha - \beta),$$

for all $y \in \Sigma$, $\alpha \in \mathbb{R}$. When $\Sigma$ is equal to $\mathbb{R}^l$ or $\mathbb{C}^l$ for $l \geq 1$, the mapping $\tau_\beta$ is assumed to be isometric with respect to Euclidean distance in $\Sigma$, for every $\beta \in \mathbb{R}$.

Theorem

Under the stated symmetry condition, the probability of decoder failure is independent of the transmitted codeword.
Symmetry Condition

For each $\beta \in \mathbb{K}$, there exists a bijection

$$
\tau_\beta : \Sigma \rightarrow \Sigma,
$$

such that the channel output probability (density) conditioned on the channel input satisfies

$$
p(y|\alpha) = p(\tau_\beta(y)|\alpha - \beta),
$$

for all $y \in \Sigma$, $\alpha \in \mathbb{K}$. When $\Sigma$ is equal to $\mathbb{R}^l$ or $\mathbb{C}^l$ for $l \geq 1$, the mapping $\tau_\beta$ is assumed to be isometric with respect to Euclidean distance in $\Sigma$, for every $\beta \in \mathbb{K}$.

**Theorem**

Under the stated symmetry condition, the probability of decoder failure is independent of the transmitted codeword.

**Examples:** $q$-ary PSK over AWGN (with $\mathbb{K}$ cyclic); orthogonal modulation over AWGN; discrete memoryless $q$-ary symmetric channel.
A linear-programming pseudocodeword of $C$ is a pair $(\mathbf{h}, \mathbf{z})$ where $\mathbf{h} \in \mathbb{R}^{(q-1)n}$ and $\mathbf{z} = (z_j, b)_{j \in J, b \in C_j}$
A linear-programming pseudocodeword of $\mathcal{C}$ is a pair $(h, z)$ where $h \in \mathbb{R}^{(q-1)n}$ and $z = (z_{j, b})_{j \in \mathcal{J}, b \in \mathcal{C}_j}$ where $z_{j, b}$ is a nonnegative integer for all $j \in \mathcal{J}$, $b \in \mathcal{C}_j$, and such that the following constraints are satisfied:

$$
\forall j \in \mathcal{J}, \forall i \in \mathcal{I}_j, \forall \alpha \in \mathbb{R}^-, h_i^{(\alpha)} = \sum_{b \in \mathcal{C}_j, b_i = \alpha} z_{j, b},
$$

and

$$
\forall j \in \mathcal{J}, \sum_{b \in \mathcal{C}_j} z_{j, b} = M,
$$

$M$ is a nonnegative integer independent of $j$. 

Mark F. Flanagan

LP Decoding of Nonbinary Linear Codes
Theorem

Assume that the all-zero codeword was transmitted.

1. If the LP decoder fails, then there exists some LP pseudocodeword \((h, z), h \neq 0\), such that \(\lambda(y)h^T \leq 0\).
Assume that the all-zero codeword was transmitted.

1. If the LP decoder fails, then there exists some LP pseudocodeword \((h, z)\), \(h \neq 0\), such that \(\lambda(y)h^T \leq 0\).

2. If there exists some LP pseudocodeword \((h, z)\), \(h \neq 0\), such that \(\lambda(y)h^T < 0\), then the LP decoder fails.
A *graph-cover pseudocodeword* is a combinatorial object which has ties to decoder failure for belief propagation decoding.

**Theorem**

There exists an LP pseudocodeword \((h, z)\) for the code \(C\) if and only if there exists a graph-cover pseudocodeword with the same pseudocodeword matrix representation.
Comparison With ML Decoding

WER and SER for the $(11, 6, 5)$ ternary Golay code under 3-PSK modulation. LP decoding is compared with the exact result for ML hard-decision decoding and the union bound for ML soft-decision decoding.

Mark F. Flanagan

LP Decoding of Nonbinary Linear Codes
Comparison With ML Decoding

WER and SER for the $(11, 6, 5)$ ternary Golay code under 3-PSK modulation. LP decoding is compared with the exact result for ML hard-decision decoding and the union bound for ML soft-decision decoding.
We have generalized the framework and theorems of [Feldman Wainwright Karger ’05] to nonbinary codes.
Conclusion

- We have generalized the framework and theorems of [Feldman Wainwright Karger ’05] to nonbinary codes
- Lays the groundwork for analyzable LDPC coded modulation
Conclusion

- We have generalized the framework and theorems of [Feldman Wainwright Karger ’05] to nonbinary codes.
- Lays the groundwork for *analyzable* LDPC coded modulation.
- Decoder failure characterized in terms of pseudocodewords.
Conclusion

- We have generalized the framework and theorems of [Feldman Wainwright Karger ’05] to nonbinary codes
- Lays the groundwork for *analyzable* LDPC coded modulation
- Decoder failure characterized in terms of pseudocodewords
- Link between LP pseudocodewords and graph-cover pseudocodewords established
Conclusion

- We have generalized the framework and theorems of [Feldman Wainwright Karger ’05] to nonbinary codes
- Lays the groundwork for analyzable LDPC coded modulation
- Decoder failure characterized in terms of pseudocodewords
- Link between LP pseudocodewords and graph-cover pseudocodewords established
- Channel symmetry condition identified for which decoder performance is independent of transmitted codeword
We have generalized the framework and theorems of [Feldman Wainwright Karger ’05] to nonbinary codes

Lays the groundwork for analyzable LDPC coded modulation

Decoder failure characterized in terms of pseudocodewords

Link between LP pseudocodewords and graph-cover pseudocodewords established

Channel symmetry condition identified for which decoder performance is independent of transmitted codeword

Simulation-based comparison with ML Decoding shows promising performance